1. Background

Unit loads are the form by which most industrial and consumer products are stored, shipped, and distributed. Exclusive of the product itself, the unit load portion of the supply chain consists of three basic components

- Packaging
- Pallets
- Unit load handling equipment

The reaction of the products to the rigors of the movement through supply chains, from manufacturer to the customer, is a consequence of how these components mechanically interact. These mechanical interactions are both static and dynamic. The interface between the packaged product and the stresses exerted by the unit load handling, storage, and shipping devices is most often a pallet. Therefore, most of these dynamic and static stresses pass through the pallet prior to exposure of the package and product. When designing distribution packaging for unitized handling, storage, and shipping, the compression strength of the packaging is a major design criteria. Compression occurs when stacking of the packaged product on pallets and the subsequent stacking of unit loads, one on top of another, during storage and transportation. Often the highest static compression stress occurs when unit loads are stack stored in warehouses or distribution centers. Under these conditions, the compression strength

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to which we design rigid packaging such as corrugated containers, plastic pails, drums, bottles, etc., becomes a function of the bearing area between the pallet deck and the packaging and the applied force. With a wood pallet, it is widely recognized that the bearing area is a function of deck design and specifically spacing between deck boards of the top and bottom decks of the pallet. What is not well understood is how the stiffness of the pallet decks affects the bearing area and consequently the compression stress on the packaging and its contents. The potential for non-uniform stress distributions and stress concentrations at the interface between the pallet deck and packaged product is shown schematically in Figure 1 and is associated with deflection of the pallet deck. Han et al. (2007) and Yoo (2008) showed that the pallet deck deforms under load and this reduces significantly the effective bearing area. Using topographical mapping techniques and FEA modeling, they predicted stress amplification factors of five to six times the average applied stress. Recently, it has been discovered, that with some modification, the engineering principles of an elastic beam supported by an elastic foundation, can be used to predict the compression stress distributions at the pallet deck/packaging interface (Yoo, 2011). White et.al., (2012) presented the results of this research at the ISTA TransPack conference in 2012. The purpose of this paper is to demonstrate how this research can be applied and used to improve the operational efficiency of supply chains.

2. Model Development

First, a brief revue of the science. To apply the principles of a beam supported by a deformable elastic foundation to the interface between a pallet deck and distribution packaging, the beam shall represent the pallet deck and the foundation shall represent the packaging and its contents. As shown in Figure 2, the actual situation within a unit load has been inverted for this application. The pallet section represents a deck board spanning two pallet stringers segments. General solutions for the beam deflection when supported on a deformable elastic foundation of stiffness, k, are shown in Figure 3.
Figure 1. Schematic diagram showing locations of compression stress concentrations associated with deflection of a pallet deck within a unit load.
General solution for beam deflection $y$ at any point $x$

$$y(x) = y_0 F_1(\lambda x) + \frac{1}{\lambda} \theta_0 F_2(\lambda x) - \frac{1}{\lambda^2 EI} M_0 F_3(\lambda x) + \frac{1}{\lambda^3 EI} Q_0 F_4(\lambda x)$$

**Figure 2.** Conceptual application of a beam supported by a deformable elastic foundation to a package supported by a pallet deck.
Where:

\[ F_1(\lambda x) = \cosh \lambda x \cos \lambda x \]
\[ F_2(\lambda x) = \frac{1}{2} (\cosh \lambda x \sin \lambda x + \sinh \lambda x \cos \lambda x) \]
\[ F_3(\lambda x) = \frac{1}{2} (\sinh \lambda x) \]
\[ F_4(\lambda x) = \frac{1}{4} (\cosh \lambda x \sin \lambda x - \sinh \lambda x \cos \lambda x) \]
\[ \lambda = \sqrt{\frac{k}{4EI}} \]

k packaging stiffness
El pallet deck stiffness
\( y_0 \) deflection at \( x = 0 \)
\( \theta_0 \) slope at \( x = 0 \)
\( M_0 \) moment at \( x = 0 \)
\( Q_0 \) shear force at \( x = 0 \)

Figure 3. Generalized solution for the deformation of an elastic beam (pallet deck supported by two stringers) supported by a uniformly distributed deformable elastic foundation (packaged product).

The force “P” is the reaction at the location of the pallet deck supports to the mass of the elastic foundation. Elastic response is assumed and has been confirmed by Weigel et al., 1999, at low levels of packaging and pallet deformation. However, theory assumes the ends of the beams are unrestrained. Pallet decks are attached to stringer or block spacers and therefore represent a range of connection stiffness or fixity. The effect of this connection fixity can be modeled using springs of appropriate stiffness and appropriate degrees of freedom and then by summing the moments around critical locations in these connections. The critical location is the inside edge of the pallet stringer or block. The solutions for a semi-rigid connection are shown in Figure 4. The moment \( M_A \) is based on a force exerted by the fastener and the distance “d” to the
\[ F_N = k y \]

\[ K = \text{rotational modulus (lbs./in.)} \]

\[ y = \text{the vertical displacement of the deck at the nail location (in.)} \]

\[ M_A = F_N \times d \]

\[ F_N = \text{force exerted by nails} \]

\[ d = \text{distance between nail and stringer} \]

**Boundary Conditions**

- (a) At \( x_2 = 0 \); \( Q_0 = P \)
- (b) At \( x_1 = 0 \) and \( x_2 = 0 \); \( M(X_1) = M(X_2) = M_0 \)
- (c) At \( X_1 = 0 \) and \( X_2 = 0 \); \( y(X_1) = y(X_2) = y_0 \)
- (d) At \( x_2 = L_2 \); \( M = M_0 \)
- (e) At \( x_2 = L_2 \); \( Q = -P \)

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**Figure 4.** Modification of the beam theory to reflect the stiffness or semi-rigid connections in pallets between deck boards and stringers.
Inside edge of the stringer. This is determined from a measured joint rotation modulus. This method of modeling the stiffness of semi-rigid nailed connections in pallets has been validated by Samarasinghe (1987), among other researchers. The model inputs are the elastic modulus (MOE) of the pallet deck, the joint rotation modulus, (K), for semi rigid connections, and the packaging stiffness (k). From the model predictions of packaging deformation along the length of the interface and the stiffness of the packaging, the force along the beam at the interface can be calculated using Hook’s Law. The stresses are assumed to vary symmetrically and in only one geometric dimension along the length of the pallet deck component.

3. Model Validation

For a detailed description of the model validation the reader is referred to White et, al., (2012) and Yoo (2011). To validate the model, a common pallet and packaging configuration was used and simulated. This included a corrugated container supported by a wood pallet section. A pallet section is a single pallet deck board span between two stringer segments. The deck spanning the stringer segments was 18 inches long and 3.5 inches wide.

A range of pallet deck stiffness was represented by two different thicknesses (0.375 and 0.750 inches) of deck components and a range of elastic moduli. Three levels of joint fixity were also incorporated. The corrugated container was an RSC style, measuring 15.5 L x 7.75 W, and 10 D inches, externally. The board grade was 42, 33C, 42. The package stiffness varied in accordance with its contents, empty, bottles, or flower sacks.

Two pressure sensor mats were used to directly measure the pressure between the pallet deck and the corrugated container. The sensitivity of the pressure sensors was 0-5 and 0-30 psi. A strain gage sensor mat is shown in Figure 5. (The sensel size was 0.25 inch square.) The test setup for measuring the compression stress at the interface between the package and the pallet deck is shown in Figure 6. The load was applied using a 10 kip servo-hydraulic MTS test machine to a level of 6 to 7 pounds per square inch.
Figure 7 is a typical visual representation of the stress distribution between the packaging and pallet deck for both sensors, as a function of package and pallet deck stiffness. The red denotes high stress and the black, zero stress. It is clear that the compression stresses are high over the stringer segments and very low between these areas of deck support.

Figures 8 are typical plots of the measured and predicted compression stresses for the different connection fixities, packaging contents, and pallet section designs. The average applied stress during the test varied from 6.25 to 6.82 psi. The bold solid curves are predicted stress and the points are measured stress. The dotted curves represent the 95% confidence boundaries. The agreement between predicted and measured is good. The maximum predicted compression stresses were as high as 50 psi over the pallet stringers while packaging between the stringers is under negligible compression stress.

Figure 5. Photograph of the strain gage pressure sensor mat used to measure the pressure distribution between the package and the pallet deck (sensel dimension 0.25 x 0.25 inches).
Figure 6. A photograph showing the test set-up for measuring the compression stress distribution between packaging and the deck of the pallet section.

Figure 7. Visual representation of the measured compression stress distribution between the packaging and pallet deck.
Figure 8. Correlation between measured and predicted compression stresses at the interface between a 16 inch long corrugated container containing plastic bottles and a wood pallet deck board with semi-rigid connections to two stringers. Maximum compression stress is over the stringer and minimum compression stress in between stringers.

The following trends are evident from both measured and predicted stresses. Compression stress on the packaging, increases as the packaging stiffness increases. However, the
compression stress is inversely related to the pallet deck stiffness. The connection fixity significantly influences pallet deck stiffness. According to these test results, the manipulation of the pallet deck design can significantly alter the effective bearing area and consequently alter the maximum compression stress to which the packaged product is exposed when unitized and moved through supply chains. The concept is to use the pallet to reduce the stress on the packaging and then reduce the cost of packaging.

4. Commercial application examples:

Since this research development, a new supply chain simulator (Best Load™), with user friendly interfaces, has been created which models the compression and bending stress interactions between packaged products, pallets, and unit load handling, storage and shipping practices¹. The following are examples of how the principles of beam on an elastic foundation can be used to reduce packaging cost and improve supply chain operational efficiencies.

4.1 Plastic pails of coating materials

Figure 9 shows the unit load of coating material, in 90 mil, 5 gallon, HDPE, plastic, pails. The maximum compression stress on the pails occurs when these unit loads are stacked stored in a warehouse, three unit loads high. Each pail contains 45.8 pounds of coating material. The total weight of the product and packaging on each pallet was 1649 pounds. The lower diagram is a histogram showing that the bottom pails on top of the bottom unit load are under greatest compression. The Best Load™ analysis further shows that the bottom pail in the six interior pail columns are under the most compression. The plot in the upper right shows that the interior edge of the pail, on the current pallet is supporting 22.06 psi of compression.

¹. Best Load™ Supply Chain Simulator, www.whiteandcompany.net
Figure 9 Compression stress analysis of the original 5 gallon pail unit load of coating material

A column of the current 90 mil, 5 gallon plastic pails (filled), were tested in compression as shown in Figure 10. The average load at failure was 2190 pounds. An alternative, lower cost, 75 mil, 5 gallon pail was similarly tested and the average load at failure was 1604 pounds. This is 27% less resistant to compression. Therefore, as a first approximation of a pallet deck to safely support product in the 75 mil pail, the compression stress must be reduced from the current 22.06 psi to less than 15 psi. To accomplish this, the pallet deck must be stiffened. Since the pallet
boards act as beams, it was decided to reduce the span between stringers by adding a fourth
stringer and by recessing the outer stringers 1 inch. This is shown in Figure 11. To reduce pallet
cost the stringer width was reduced from 1.5 inches to 1.25 inches. The structural analysis using
Best Load™ of this new pallet, predicts a maximum compression on the pails of 13.70 psi.
Prototypes of the proposed unit load with the new pallet design and 75 mil pails were tested in the
field to verify performance. Within three months the new unit load designs were implemented.
Figure 11 Structural analysis of the prototype unit load of 75 mil plastic pails on the proposed 4 stringer winged pallet

The cost per pallet increased by $0.98 and the cost per pail decreased by $0.31. The net savings per unit load was $10.11. This was an annual packaging spend reduction of $209,199.00

4.2 Twenty four packs of bottled water

The stacking patterns on pallets can significantly affect the compression stresses on packaged product. Figure 12 is a Best Load™ analysis of two stacking patterns of shrink wrapped, 24 packs of 16.9 oz. bottles of water on a block class wood pallet. The stresses are greatest on the bottom layer of 24 packs. Notice the top two layers are interlocked for stability and the unit load is stretch
wrapped. A change in stacking pattern alters the maximum compression stress by 29%. This is a potential reduction of maximum compression stress from 10.2 to 7.2 psi. Whether manually or automatically stacking product on pallets such changes in patterns have little or no effect on the cost of assembling unit loads. **Therefore this potential reduction in compression stress is free and will significantly reduce leakers and related damage.**

![Diagram of shrink-wrapped pallets](image)

### 5.0 Summary

Unit load supply chains have three primary components; the pallet, layers of packaging, and equipment used to move, store, and ship unit loads. Today these components are typically designed by vendors with little interaction between designers. This **Component Design Process** has led to supply chains operating with significant avoidable cost. To improve the safety, sustainability, and efficiency of these supply chains, designers must make a fundamental shift to a **Systems Design Process**. To accomplish this we need to research how these components...
mechanically interact and to educate and train a new population of logistics and packaging professionals, knowledgeable in the design of all components. These new professionals must be provided tools that model how the components interact. In this paper the process of systems based design of supply chains has been demonstrated using a new supply chain simulator Best Load™

References


Samarasinghe, S., 1987, Predicting the Rotation Modulus for Block Pallet Joints. M.S. Thesis, Department of Wood Science and Forest Products, Virginia Tech, Blacksburg, VA.


